

Geometric Properties Of A New Subclass Of Analytic Functions By Means Of Chebyshev Polynomials

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Abstract— In this work, Opoola Differential Operator which is a generalization of Salagean Differential Operator and Al Oboudi Differential Operator is used to define new subclasses of analytic univalent functions using subordination principle. Also, by means of Chebyshev polynomials of the second kind, coefficient estimates, upper bounds for the Fekete-Szego functional for the new subclasses defined are obtained.

Keywords— Analytic, Univalent, Opoola differential operator, Chebyshev polynomials, coefficient estimates, Fekete-Szego functional.

Introduction

Let A be the class of functions analytic in the unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. $f(z)$ is said to be in the class S if

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k,$$

with the conditions $f(0) = 0$, $f'(0) = 1$.

Chebyshev polynomials which are of four kinds, are widely used in many areas like numerical solutions of ordinary and partial differential equations, analytic functions and so on. Many research journals are using the first and second kinds of Chebyshev polynomials because of its orthogonality property. See [2-4], [9] and [12]. The Chebyshev polynomial of the second kind in

of degree n can be defined as:

$$U_n(t) = \frac{\sin(n+1)\theta}{\sin\theta}, \quad t = \cos\theta, \quad t \in [-1, 1]$$

The generating function for Chebyshev polynomial of the second kind is given as

$$H(z, t) = 1 + 2tz + (4t^2 - 1)z^2 + (8t^3 - 4t)z^3 + (16t^4 - 12t^2 + 1)z^4 + \dots$$

The Fekete-Szego inequality states that if

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots$$

is a univalent analytic function on the unit disk and

$0 \leq \lambda \leq 1$, then

$$C(\lambda) = |a_3 - \lambda a_2^2| < 1 + 2e^{\frac{-2\lambda}{1-\lambda}}$$

For a class of functions in A and a real number the Fekete-Szego problem is all about finding the best possible for functions in A . Some of the researchers that have worked in this area are: [10], [13] and [20]. Let $f(z)$ and $g(z)$ be analytic functions in the unit disk, then $f(z)$ is subordinate to $g(z)$ if it can be written as $f(z) = g(w(z))$, if there exists a function $w(z)$ analytic in U with $w(0)=0$, $|w(z)| < 1$ which is called the Schwarz function such that $f(z) = g(w(z))$. If the function g is univalent in U , then $f(z) = g(w(z))$; $z f(0) = g(0)$ and $f'(0) = g'(0)$. see [16]. A function of the form (2) is star-like with respect to the unit disk if it maps a unit disk onto a star-like domain and it is denoted by A^* . A necessary and sufficient condition for f to be star-like is that

$$\operatorname{Re} \left\{ z \frac{f'(z)}{f(z)} \right\} > 0, \quad z \in U$$

An analytic function is convex if it maps conformally onto a convex domain and it is denoted by C . Equivalently, if and only if it satisfies the following condition

$$\operatorname{Re} \left\{ 1 + z \frac{f''(z)}{f'(z)} \right\} > 0, \quad z \in U$$

II Preliminaries

Lemma 1. [6]

If $p \in \mathcal{P}$, with the series expansion $p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \dots, z \in U$, with $\operatorname{Re}(p(z)) > 0$ and $p(0) = 1$, Then $|p_n| \leq 2, n = \{1, 2, 3, \dots\}$.

Lemma 2. [14]

Let the function be given by the series (2). Then for any complex number

$$|p_2 - \sigma p_1^2| \leq 2 \max \{1, |2\sigma - 1|\}$$

And the result is sharp for the functions given by

$$p(z) = \frac{1+z}{1-z} \quad (z \in \mathbb{U}).$$

Lemma 3. [15]

If $p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \dots$ is analytic function with positive real parts in \mathbb{U} , then

$$|p_2 - v p_1^2| \leq \begin{cases} -4v + 2, & \text{if } v \leq 0 \\ 2, & \text{if } 0 \leq v \leq 1 \\ 4v - 2, & \text{if } v \geq 1 \end{cases}$$

Opoola Differential Operator. [19]: For $p \geq 0, 0 \leq \mu \leq \beta, n \in \mathbb{N}_0, z \in \mathbb{U}$, the Opoola differential operator $D^n(p, \mu, \beta)f: A \rightarrow A$ is defined as follows

$$D^0(p, \mu, \beta)f(z) = f(z)$$

$$D^1(p, \mu, \beta)f(z) = pz f'(z) - z(\beta - \mu)p + (1 + (\beta - \mu - 1)p)f(z)$$

$$D^n(p, \mu, \beta)f(z) = (D(D^{n-1}(p, \mu, \beta)f(z))) = z + \sum_{k=2}^{\infty} [1 + (k + \beta - \mu - 1)p]^n a_k z^k \quad (3)$$

This operator was studied by the author in [8].

Remark 1:

- When $p = 1, \mu = \beta$, then $D^n(p, \mu, \beta)f(z)$ becomes the Salagean differential operator.[22]
- When $\mu = \beta$, then $D^n(p, \mu, \beta)f(z)$ becomes the Al-Oboudi differential operator.

Definition: For a function given by (2), analytic in the unit disk is in the class $SC(n, b, \mu, \beta, p, U_n(z))$ if it satisfies the subordination

$$1 + \frac{1}{b} \left\{ \frac{z(D^n(\mu, \beta, p)f(z))'}{D^n(\mu, \beta, p)f(z)} - 1 \right\} < H(z, t) \quad (4)$$

Where $n \in \mathbb{N}_0 = \{0\} \cup \{1, 2, 3, \dots\}, b \in \mathbb{C} \setminus \{0\}, D^n(\mu, \beta, p)f(z)$ is

$H(z, t)$ is given by eq.(1).

given by equation (3) and

Remark 2 :

when $H(z, t)$ is replaced with $\phi(z)$ (see [15]) in equation (4), then the following remarks hold

- When equation (4) becomes the class Which implies that introduced by [21].
- When , , equation (4) become the classes of functions known as Ma- Minda star-like and convex functions respectively. Hence, See [15].
- When equation (4) becomes the class introduced by [21].

III Results And Discussions

Coefficient Bounds For Functions In The Class $SC(n, b, \mu, \beta, p, U_n(t))$

Theorem 1: Let the function $f \in A$ given by (2). If the function f is in the class $SC(n, b, \mu, \beta, p, U_n(t))$, $t \in (0, 1]$, $\theta \in [0, \pi]$, then

$$\begin{aligned}
 |a_2| &\leq \frac{2t|b|}{[1 + (\beta - \mu + 1)p]^n} \\
 |a_3| &\leq \frac{1}{[1 + (\beta - \mu + 2)p]^n} \left| bt - \frac{1}{2}(b(2t - 4t^2 + 1) - b^2 4t^2) \right| \\
 |a_4| &\leq \frac{1}{3[1 + (\beta - \mu + 3)p]^n} |t^3\{8b + 4b^3 + 12b^2\} - t(4b + 3b^2)| \\
 |a_5| &\leq \frac{1}{4[1 + (\beta - \mu + 4)p]^n} \left| t^4(72b^2 - 16b + 24b^4) \right. \\
 &\quad \left. + t^3(40b^2 + 48b - 32b^3) - t^2(76b^2 + 12b + 144b^3) \right. \\
 &\quad \left. - t(24b - 18b^2) - 36b^3 + \frac{37b^2}{2} + 5b \right| \quad (5)
 \end{aligned}$$

Proof:

If $f(z) \in SC(n, b, \mu, \beta, p, U_n(t))$, then there exist a Schwarz function $\omega(z)$, analytic in the open unit disk

\mathbb{U} with $\omega(0) = 0, |\omega(z)| < 1$ such that from the definition of subordination and equation(4)

$$1 + \frac{1}{b} \left\{ \frac{z(D^n(\mu, \beta, p)f(z))'}{D^n(\mu, \beta, p)f(z)} - 1 \right\} = H(\omega(z), t)$$

From the definition of Chebyshev polynomial,

$$H(\omega(z), t) = 1 + U_1(t)(\omega(z)) + U_2(t)(\omega(z))^2 + U_3(t)(\omega(z))^3 + U_4(t)(\omega(z))^4 + \dots$$

Since $\omega(z)$ is a Schwarz function then the function $p(z)$ is defined as

$$p(z) = \frac{1 + \omega(z)}{1 - \omega(z)} = 1 + p_1(z) + p_2 z^2 + \dots$$

Which implies that

$$\omega(z) = \frac{p_1 z}{2} + \frac{1}{2} \left(p_2 - \frac{p_1^2}{2} \right) z^2 + \frac{1}{2} \left(p_3 - p_1 p_2 + \frac{p_1^3}{4} \right) z^3 + \frac{1}{2} \left(p_4 - p_1 p_3 - \frac{p_2^2}{2} + \frac{3p_1^2 p_2}{4} - \frac{p_1^4}{8} \right) z^4 + \dots$$

Hence,

$$\begin{aligned}
 H(\omega(z), t) = & \frac{U_1((t)p_1 z}{2} + \left[\frac{U_1((t))}{2} \left(p_2 - \frac{p_1^2}{2} \right) + U_2((t)) \frac{p_1^2}{4} \right] z^2 \\
 & + \left[\frac{U_1((t))}{2} \left(p_3 - p_1 p_2 + \frac{p_1^3}{4} \right) + \frac{U_2((t))}{2} \left(p_1 p_2 - \frac{p_1^3}{2} \right) + U_3((t)) \frac{p_1^3}{8} \right] z^3 \\
 & + \left[\frac{U_1((t))}{2} \left(p_4 - p_1 p_3 - \frac{p_1^2}{2} + \frac{3p_1^2 p_2}{4} - \frac{p_1^4}{8} \right) + \frac{U_2((t))}{2} \left(p_1 p_3 + \frac{p_1^2}{2} - \frac{3p_1^2 p_2}{4} - \frac{3p_1^4}{8} \right) + \frac{U_3((t))}{8} \left(3p_1^2 p_2 - \frac{3p_1^4}{2} \right) \right. \\
 & \left. + \frac{U_4((t)p_1^4}{16} \right] z^4 + \dots \quad (5)
 \end{aligned}$$

Also,

$$\begin{aligned}
 & 1 + \frac{1}{b} \left\{ \frac{z(D^n(\mu, \beta, p)f(z))'}{D^n(\mu, \beta, p)f(z)} - 1 \right\} \\
 & = \frac{1}{b} \{ [1 + (\beta - \mu + 1)p]^n a_2 z + (2[1 + (\beta - \mu + 2)p]^n a_3 - [1 + (\beta - \mu + 1)p]^{2n} a_2^2) z^2 \\
 & + (3[1 + (\beta - \mu + 3)p]^n a_4 - 3[1 + (\beta - \mu + 1)p]^n [1 + (\beta - \mu + 2)p]^n a_2 a_3 \\
 & + [1 + (\beta - \mu + 2)p]^{2n} a_2^2) z^3 \\
 & + (4[1 + (\beta - \mu + 4)p]^n a_5 - 4[1 + (\beta - \mu + 1)p]^n [1 + (\beta - \mu + 3)p]^n a_2 a_4 \\
 & - 2[1 + (\beta - \mu + 1)p]^{2n} a_3^2 + 4[1 + (\beta - \mu + 1)p]^{2n} [1 + (\beta - \mu + 2)p]^n a_2^2 a_3 \\
 & - [1 + (\beta - \mu + 1)p]^{4n} a_2^4) z^4 + \dots \} \quad (6)
 \end{aligned}$$

Comparing the coefficients of the powers of z in equations (5) and (6) and using lemma 1, the result of equation follows:

Corollary:1

$$f(z) \in SC(0, b, \mu, \beta, p, U_n(t)), t \in (0, 1], \text{ then } |a_2| \leq 2t|b|$$

$$|a_3| \leq 2t^2|(b + b^2)| + \frac{1}{2}$$

$$|a_4| \leq \frac{1}{3} |t^3\{8b + 4b^3 + 12b^2\} - t(4b + 3b^2)|$$

$$|a_5| \leq \frac{1}{4} \left| t^4(72b^2 - 16b + 24b^4) + t^3(40b^2 + 48b - 32b^3) - t^2(76b^2 + 12b + 144b^3 - t(24b - 18b^2) - \right.$$

$$\left. + \frac{37b^2}{2} + 5b \right|$$

Corollary 2:

When $SC(n, b, \mu, \beta, p, U_n(t))$ is reduced to $S(0, b, \phi(z))$, studied in [21].

We proceed to solve the Fekete-Szegő problem for the new class of function defined when σ is complex

Fekete-Szegő Inequality For Functions In The Class

$SC(n, b, \mu, \beta, p, U_n(t))$.

Theorem 2: If $f(z)$ given by equation (2) belongs to the class of functions $SC(n, b, \mu, \beta, p, U_n(t))$, then for any complex number $\sigma, t \in (0, 1], \theta \in [0, \pi]$

$$|a_3 - \sigma a_2^2| \leq \frac{bt}{[1 + (\beta - \mu + 2)p]^n} \max \left[1, \left| \frac{4t^2 - 1}{2t} + \left(2\sigma \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right| \right]$$

$$b \in \mathbb{C} \setminus \{0\}, \quad 0 \leq \mu \leq \beta, \quad p \geq 0, \quad \sigma \in \mathbb{C}$$

Proof:

From equation (5), written as the Fekete-Szego inequality can be

$$|a_3 - \sigma a_2^2| \leq \left| \frac{bt}{2[1 + (\beta - \mu + 2)p]^n} \{p_2 - v p_1^2\} \right|$$

$$\text{Where } v = \frac{1}{2} \left\{ 1 - \frac{4t^2 - 1}{2t} + \left(2\sigma \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right\} \text{ On}$$

application of lemmas 1 and 2, the result follows.

Corollary 3: .

If $f(z) \in SC(b, \phi(z))$, then

It reduces the class of functions studied in [21]. Next, we consider the case when is real in the following theorem

Theorem 3: Let $b > 0, t \in (0, 1]$ and let $f(z)$ be given by equation (2) belongs to the class of functions $SC(n, b, \mu, \beta, p, U_n(t))$. Then for $\delta \in \mathbb{R}$, we have

$$|a_3 - \delta a_2^2| \leq \begin{cases} k \left[-2 \left\{ 1 - \frac{4t^2 - 1}{2t} + (2\delta q - 1)2bt \right\} + 2 \right] & \text{if } \delta \leq \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\} \\ 2k & \text{if } \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\} \leq \delta \leq \frac{1}{2q} \left\{ \frac{4t^2 + 2t - 1}{4bt^2} + 1 \right\} \\ k \left[2 \left\{ 1 - \frac{4t^2 - 1}{2t} + (2\delta q - 1)2bt \right\} - 2 \right] & \text{if } \delta \geq \frac{1}{2q} \left\{ \frac{4t^2 + 2t - 1}{4bt^2} + 1 \right\} \end{cases}$$

$$\text{Where } k = \frac{1}{2[1 + (\beta - \mu + 2)p]^n}$$

Proof:

Let $\delta \leq \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\}$ then from equation (5), the Fekete-Szegő inequality can be written as

$$|a_3 - \sigma a_2^2| \leq \left| \frac{bt}{2[1 + (\beta - \mu + 2)p]^n} \{p_2 - v p_1^2\} \right|$$

$$\text{Where } v = \frac{1}{2} \left\{ 1 - \frac{4t^2 - 1}{2t} + \left(2\delta \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right\}$$

On application considered of lemma 3, the following cases are

Case 1:

$|a_2 - v a_1^2| \leq -4v + 2$, if $v \leq 0$ this implies that

$$|a_2 - v a_1^2| = -4v + 2 = -4 \left\{ \frac{1}{2} \left\{ 1 - \frac{4t^2 - 1}{2t} + \left(2\delta \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right\} \right\} + 2$$

That is when

$$\frac{1}{2} \left\{ 1 - \frac{4t^2 - 1}{2t} + \left(2\delta \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right\} \leq 0,$$

Let $\frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} = q$, then

$$|a_3 - \delta a_2^2| \leq k \left[-2 \left\{ 1 - \frac{4t^2 - 1}{2t} + (2\delta q - 1) 2bt \right\} + 2 \right] \text{ if } \delta \leq \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\} \quad (8)$$

Case 2:

$$|a_2 - va_1^2| \leq 2, \text{ if } 0 \leq v \leq 1$$

If $0 \leq v$, it implies that

$$\frac{1}{2} \left\{ 1 - \frac{4t^2 - 1}{2t} + \left(2\delta \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right\} \geq 0$$

$$\text{Then } \delta \geq \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\}$$

And if $v \leq 1$, it implies that

$$\frac{1}{2} \left\{ 1 - \frac{4t^2 - 1}{2t} + \left(2\delta \frac{[1 + (\beta - \mu + 2)p]^n}{[1 + (\beta - \mu + 1)p]^{2n}} - 1 \right) 2bt \right\} \leq 1$$

Which also implies that

$$\delta \leq \frac{1}{2q} \left\{ \frac{4t^2 + 2t - 1}{4bt^2} + 1 \right\}$$

So,

$$|a_3 - \delta a_2^2| \leq 2k \text{ if } 0 \leq \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\} \leq \delta \leq \frac{1}{2q} \left\{ \frac{4t^2 + 2t - 1}{4bt^2} + 1 \right\} \quad (9)$$

Case 3:

$$|a_2 - va_1^2| \leq 4v - 2, \text{ if } v \geq 1$$

then

$$|a_3 - \delta a_2^2| \leq k \left[2 \left\{ 1 - \frac{4t^2 - 1}{2t} + (2\delta q - 1) 2bt \right\} - 2 \right] \text{ if } \delta \geq \frac{1}{2q} \left\{ \frac{4t^2 - 2t - 1}{4bt^2} + 1 \right\} \quad (10)$$

Hence, from equations (8), (9) and (10), the result of theorem 3 holds.

Conclusion

Opoola differential equation, see [19], when some conditions are set in, is a generalization of both Salagean and Al-Oboudi differential operators. This operator was used to define a new subclass of analytic and univalent functions. New results were obtained such as the proofs of theorem 1-3 and corollary 1. Also, the generalization of the existing results such as Remarks 1 and 2, corollaries 2 and 3 were obtained in this work.

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