

Analysis of the Effects of PID Parameters on the Deflection Angle of a Frusta Pendulum

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Abstract

The Frusta pendulum is a representative nonlinear system widely used as a benchmark for evaluating control algorithms in automation and robotics. Among various control strategies, the Proportional–Integral–Derivative (PID) controller remains one of the most widely adopted owing to its simple structure and satisfactory industrial performance. However, the individual influence of the proportional, integral, and derivative gains (K_{ip} , K_{id} , and K_{id}) on the dynamic response of the Frusta pendulum has not been systematically quantified. This paper presents a quantitative investigation of these effects on the pendulum angular deviation α through mathematical modeling and MATLAB/Semolina simulations. The results show that K_{ip} primarily determines response speed but increases overshoot when excessively large; K_{id} eliminates steady-state error but may prolong settling time and degrade stability if improperly tuned; and K_{id} suppresses oscillations and improves transient performance, though excessive values reduce response speed and increase noise sensitivity. The optimized PID parameters enable a short settling time, low steady-state error, and reduced oscillations. Compared with the Linear Quadratic Regulator (LQR), the PID controller offers greater implementation simplicity and ease of tuning, whereas LQR provides smoother responses and superior stability at the cost of requiring an accurate model and more sophisticated design. This work establishes a systematic quantitative framework for evaluating individual PID parameters, identifies an effective parameter set, and provides practical guidelines for controller tuning in balancing systems such as self-balancing robots.

Keywords: Frusta pendulum; PID control; Mat lab/Semolina; Nonlinear control; Parameter tuning.

I. Introduction

The Frusta pendulum is one of the most representative nonlinear and inherently unstable systems in modern control theory. Owing to its complex kinematic and dynamic characteristics, it has been extensively employed as a benchmark platform for evaluating control algorithms in automation, robotics, mechatronics, and intelligent control systems [1].

Among the available control strategies, the Proportional–Integral–Derivative (PID) controller remains one of the most widely adopted solutions because of its simple structure, low

implementation cost, and satisfactory performance in a wide range of industrial applications. However, the control performance of a PID controller is highly dependent on the proper selection of the proportional, integral, and derivative gains (K_{ip} , K_{id} , and K_{id}). Improper parameter tuning may result in excessive overshoot, prolonged settling time, increased steady-state error, or even system instability [2]. Therefore, understanding the individual influence of each PID parameter on the dynamic behaviour of the Furuta pendulum is essential for both theoretical analysis and practical controller design.

Recent studies have investigated various approaches to improve the control performance of Furuta pendulum systems. Almeshal et al. reported that increasing the proportional gain improves the response speed but also increases system oscillations, whereas the integral and derivative gains play important roles in reducing the steady-state error and enhancing system stability [3]. CSU compared the performance of PID and Linear Quadratic Regulator (LQR) controllers and demonstrated that PID is easier to implement, whereas LQR achieves superior control performance through optimal state feedback [4]. Furthermore, Erotica et al. proposed a Fractional-Order PID controller that provides improved dynamic performance compared with the conventional PID controller for nonlinear systems [5].

Despite these advances, most existing studies report the qualitative or isolated effects of individual PID gains, without a unified simulation framework that systematically and quantitatively compares the contributions of K_{ip} , K_{id} , and K_{id} on the dynamic response of the Furuta pendulum. This research gap motivates a more comprehensive investigation into the relative contribution of each PID parameter to the overall control performance.

Accordingly, the objective of this study is to develop a mathematical model of the Furuta pendulum, investigate the influence of individual PID parameters through MATLAB/Simulant simulations, optimize the PID controller parameters, and compare the control performance with that of the LQR. The findings provide a systematic understanding of the role of each PID parameter and offer practical guidance for controller tuning and the implementation of balancing control systems in both research and engineering applications.

II. Research Methodology

A. System Modeling

The Furuta pendulum is selected as the study object because it represents a highly nonlinear and inherently unstable electromechanical system widely used for validating control algorithms. The system consists of a horizontal rotating arm driven by a DC motor, with a pendulum freely attached at its end, as illustrated in Fig. 1. Let θ denote the rotation angle of the arm and α denote the deflection angle of the pendulum from the upward vertical position.

The nonlinear dynamic model of the Furuta pendulum is derived using the Euler–Lagrange formulation. The total kinetic energy T and potential energy V of the system are expressed as:

$$T = \frac{1}{2}(J_r + m_p L_r^2)\dot{\theta}^2 + \frac{1}{2}m_p l_p^2 \dot{\alpha}^2 - m_p L_r l_p \cos\alpha \dot{\theta}\dot{\alpha} + \frac{1}{2}J_p \dot{\alpha}^2 \quad (1)$$

$$V = m_p g l_p \cos\alpha \quad (2)$$

where J_r and J_p are the moments of inertia of the arm and pendulum, respectively; m_p is the pendulum mass; L_r is the arm length; l_p is the distance from the pivot to the pendulum's center of

mass; and g is the gravitational acceleration. Applying the Euler–Lagrange equation.

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i \quad (3)$$

with $L = T - V$ and generalized coordinates $q_i = [\theta, \alpha]^T$, yields the coupled nonlinear equations of motion governing the arm and pendulum dynamics:

$$(J_r + m_p L_r^2) \ddot{\theta} - m_p L_r l_p \cos \alpha \ddot{\alpha} + m_p L_r l_p \sin \alpha \dot{\alpha}^2 + b_r \dot{\theta} = \tau_m \quad (4)$$

$$(J_r + m_p L_r^2) \ddot{\alpha} - m_p L_r l_p \cos \alpha \ddot{\theta} - m_p g l_p \sin \alpha + b_r \dot{\alpha} = 0 \quad (5)$$

where b_r and b_p are the viscous friction coefficients of the arm and pendulum, respectively, and τ_m is the driving torque generated by the DC motor, related to the applied voltage $u(t)$ through the motor's electrical and mechanical characteristics. Eqs. (4)–(5) constitute the complete nonlinear dynamic model of the Furuta pendulum, in which the pendulum deflection angle α is selected as the output variable for evaluating control performance.

B. PID Controller Design

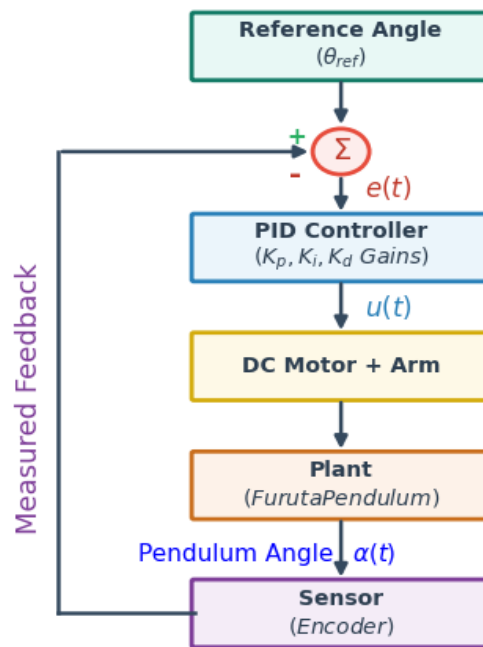


Figure 1 Block diagram of the PID-controlled Furuta pendulum system.

A PID controller is adopted to stabilize the Furuta pendulum owing to its simple structure and ease of practical implementation. The control signal is expressed as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (6)$$

where $u(t)$ is the control input signal, $e(t)$ denotes the error between the reference signal and the measured pendulum angle, K_p , K_i , and K_d are the proportional, integral, and derivative gains, respectively. Each controller parameter serves a distinct role: K_p determines the response speed

and sensitivity of the control system; K_i eliminates steady-state error through integral action; K_d enhances system stability by damping oscillations and reducing overshoot.

The initial PID gains are obtained using the Ziegler–Nichols tuning method and are subsequently fine-tuned through Matlab/Simulink simulations to achieve a fast settling time, low steady-state error, and minimal oscillatory behavior.

Fig. 1 illustrates the closed-loop control architecture adopted in this study. The reference angle is compared with the measured pendulum angle to generate the control error $e(t)$. This error signal is processed by the PID controller to produce the control input applied to the DC motor. The pendulum angle is measured using an encoder and fed back to the controller, thereby forming a closed-loop feedback control system.

C. Simulation Procedure and Performance Evaluation

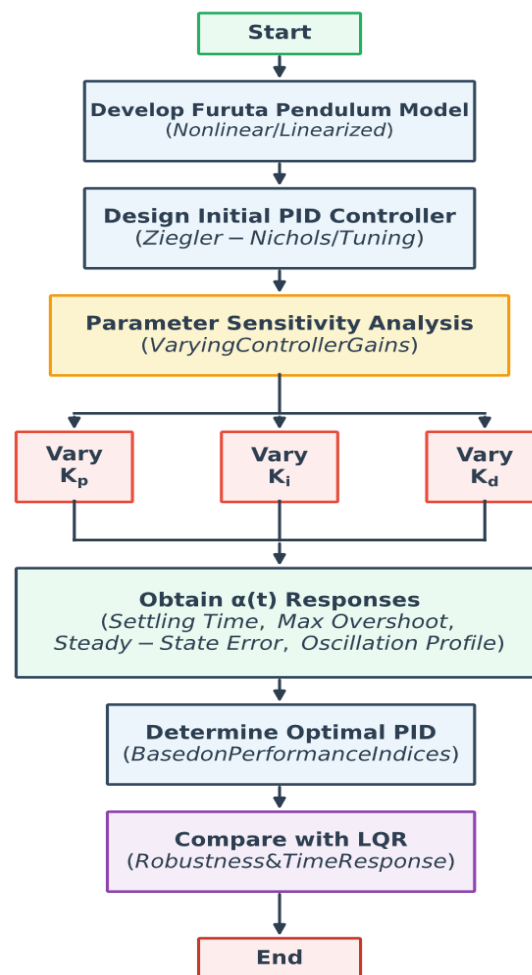


Fig. 2 Proposed research workflow.

To quantitatively investigate the influence of each PID parameter on the pendulum deflection angle, the proposed methodology consists of five sequential stages, as illustrated in Fig. 2: (I) establishing the nonlinear mathematical model; (ii) implementing the model in MATLAB/Semolina; (iii) tuning the initial PID gains using the Ziegler–Nichols method; (iv)

performing individual and combined parameter sensitivity simulations; and (v) evaluating and comparing the control performance against the LQR controller.

The dynamic performance of the control system is evaluated using four standard performance indices: Settling Time; Overshoot; Steady-State Error; Oscillation Amplitude around the equilibrium position. These indices are employed to quantify the individual effects of K_{ip} , K_{id} , and K_{id} and to determine the optimal PID parameter set. Finally, the performance of the optimized PID controller is compared with that of the LQR controller to highlight the advantages and limitations of both approaches.

III. Results And Discussion

A. Effect of the proportional gain - K_{ip}

To investigate the influence of the proportional gain, simulations were performed with $K_{id} = 0$, $K_{id} = 0$, and a sampling time of $T_s = 10^{-5}$ s, while varying the value of K_{ip} .

The simulation results indicate that increasing K_{ip} significantly improves the transient response of the Frusta pendulum. At relatively low values of K_{ip} , the system exhibits a slow response, a long settling time, and a noticeable transient error. As K_{ip} increases, the settling time decreases because the controller generates a larger corrective torque to restore the pendulum to its equilibrium position. However, a higher proportional gain also results in increased overshoot and oscillation amplitude. When K_{ip} exceeds a critical value, sustained oscillations may occur, leading to degraded system stability. This behavior can be attributed to the excessive amplification of the control error, causing the actuator to continuously overcompensate.

For the case of $K_{ip} = 15$, $K_{id} = 0$, and $K_{id} = 0$, the simulation results shown in Fig. 3 indicate that the system reaches a near-equilibrium state after approximately 0.6 s, while the pendulum angle continues to oscillate with an amplitude of approximately $\pm 8^\circ$ around the equilibrium position. These results suggest that increasing the proportional gain effectively accelerates the system response but is insufficient to suppress oscillations. Therefore, the incorporation of integral and derivative actions is required to further improve system stability and overall control performance.

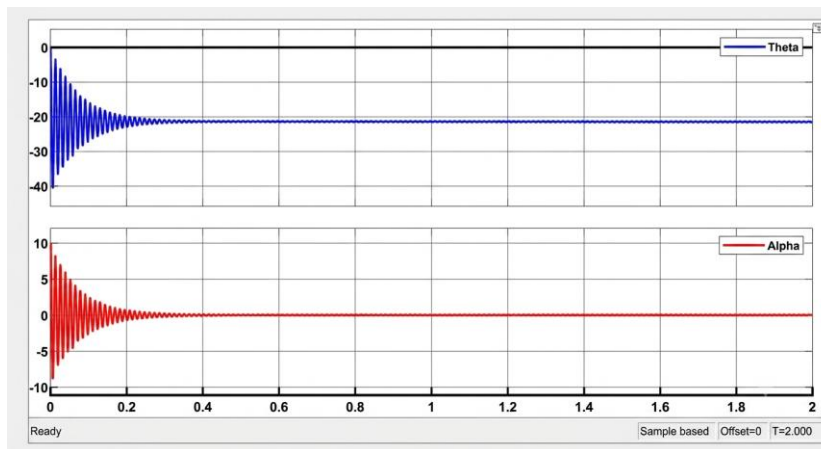


Fig. 3 Response of the pendulum angular deviation α under the influence of the proportional gain K_{ip}

B. Effect of the integral gain - K_{id}

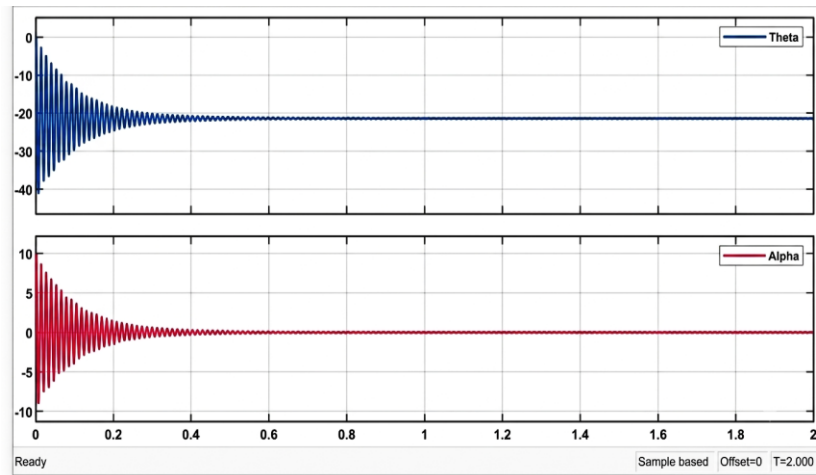


Fig. 4 Effect of the proportional gain K_{ip} and integral gain K_{id} on the pendulum angular deviation α .

To evaluate the influence of the integral gain, simulations were conducted with $K_{ip} = 15$, $K_{id} = 0$, and a sampling time of $T_s = 10^{-5}$ s, while varying the value of K_{id} .

The simulation results demonstrate that the integral gain plays a crucial role in eliminating the steady-state error of the control system. As K_{id} increases from zero to an appropriate value, the pendulum angle α converges more accurately to the equilibrium position, resulting in a significant reduction of the steady-state error. This improvement is attributed to the integral action, which continuously accumulates the control error over time and compensates for the residual error remaining after the transient response.

However, when the integral gain becomes excessively large, the system exhibits low-frequency oscillations and a prolonged settling process. This behavior is mainly caused by integral windup, where excessive accumulation of the error generates an overly large control signal, leading to repeated overcorrection's around the equilibrium position before the system eventually stabilizes.

For the case of $K_{ip} = 15$, $K_{id} = 100$, and $K_{id} = 0$, the simulation results presented in Fig. 4 indicate that the system reaches a near-equilibrium state after approximately 0.8 s. Although the steady-state error is almost completely eliminated, the pendulum continues to oscillate around the equilibrium position with gradually decreasing amplitude, requiring several oscillation cycles before achieving steady operation. These results indicate that the integral action effectively improves steady-state accuracy; however, an excessively large integral gain increases the settling time and deteriorates the overall stability of the control system. Therefore, an appropriate selection of K_{id} is essential to balance steady-state accuracy and dynamic performance.

C. Effect of the derivative gain - K_{id}

To investigate the influence of the derivative gain, simulations were performed with $K_{ip} = 15$, $K_{id} = 0$, and a sampling time of $T_s = 10^{-5}$ s, while varying the value of K_{id} .

The simulation results demonstrate that the derivative gain plays an important role in improving

the transient performance of the control system. As K_{id} increases, the controller anticipates the rate of change of the control error and introduces an appropriate damping action, thereby reducing overshoot and suppressing oscillations around the equilibrium position. Consequently, the system exhibits a smoother transient response and enhanced stability.

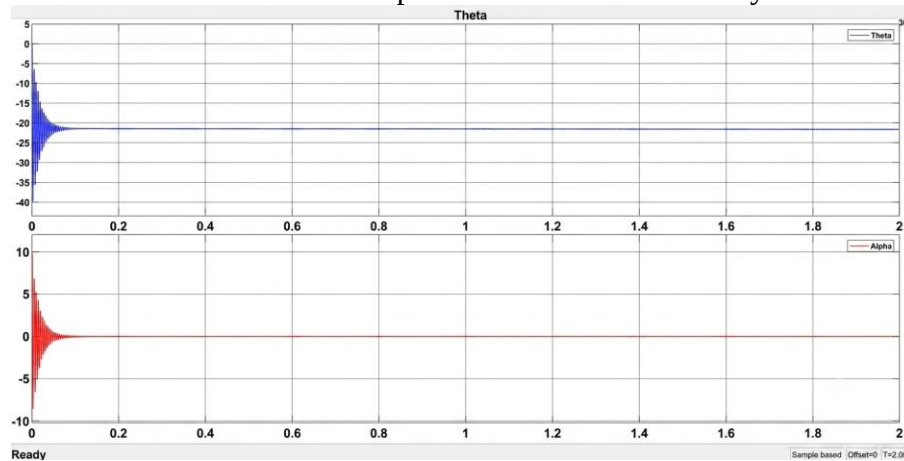


Fig. 5 Effect of the proportional gain K_{ip} and derivative gain K_{id} on the pendulum angular deviation α .

However, an excessively large derivative gain introduces excessive damping, which slows the variation of the control signal and may increase the settling time. Furthermore, in practical implementations, high values of K_{id} amplify measurement noise and sensor disturbances, potentially degrading the overall control performance.

For the case of $K_{ip} = 15$, $K_{id} = 0$, and $K_{id} = 1$, the simulation results presented in Fig. 5 indicate that the system reaches a near-equilibrium state after approximately 0.2 s. Compared with the proportional-only controller, the oscillation amplitude is reduced and the transient response becomes more stable. Nevertheless, small oscillations around the equilibrium position remain because the integral action is absent. These results indicate that the derivative term effectively enhances the transient behavior of the Frusta pendulum, whereas optimal control performance can only be achieved through the coordinated tuning of all three PID parameters.

D. Performance comparison between PID and LQR controllers

After investigating the individual effects of the proportional, integral, and derivative gains, the PID parameters were optimized using the PID Tuner in Mat lab to achieve a balance between response speed and system stability. The optimized controller parameters were obtained as $K_{ip} = 17$, $K_{id} = -0.0007$, $K_{id} = 0.1$, with a sampling time of $T_s = 10^{-5}$ s.

Table 1 Comparison between PID and LQR Controllers

Criterion	PID	LQR
Controller structure	Simple	State-feedback based
Design complexity	Low	High
Mathematical model requirement	Moderate	Accurate model required
Tuning effort	Low	Relatively high
Overshoot	Higher	Lower
Oscillation	Larger	Smaller
Practical implementation	Easy	More difficult

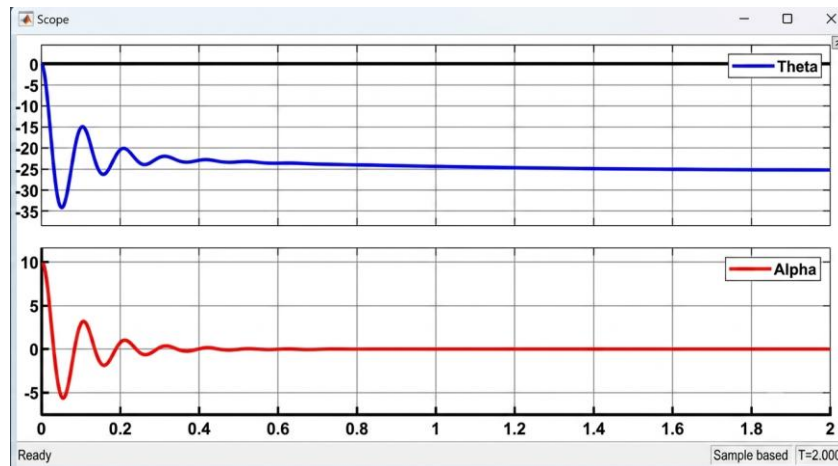


Fig.6 Pendulum angular response using the PID controller optimized by Mat lab.

The simulation results presented in Fig. 6 Indicate that the system reaches a near-equilibrium state after approximately 0.5 s, while the oscillation amplitude around the equilibrium position is significantly reduced compared with the cases where only individual PID components are considered. These results suggest that the coordinated action of the proportional, integral, and derivative terms substantially improves the transient response and provides a favorable balance between response speed, stability, and steady-state accuracy.

To further evaluate the effectiveness of the optimized PID controller, its performance was compared with that of the LQR using the same system model. The comparison is summarized in Table 1.

The comparison indicates that the PID controller offers the advantages of simple implementation, straightforward parameter tuning, and strong applicability in practical control systems. In contrast, the LQR controller provides smoother responses, lower overshoot, and improved stability by optimizing a quadratic cost function. However, these advantages are achieved at the expense of requiring an accurate mathematical model and complete state

information, which increases the implementation complexity.

Overall, for the investigated Furuta pendulum system, the optimized PID controller achieves satisfactory control performance with a short settling time, reduced oscillations, and low steady-state error while maintaining the simplicity and practicality required for real-time control applications.

E. General discussion

The obtained results demonstrate that the three PID parameters influence the dynamic behavior of the Furuta pendulum through different control mechanisms. The proportional gain K_p primarily determines the response speed of the system. Increasing K_p accelerates the transient response but simultaneously increases overshoot and oscillation amplitude. The integral gain K_i effectively eliminates the steady-state error, thereby improving steady-state accuracy. However, an excessively large integral gain may lead to error accumulation, resulting in longer settling times and degraded system stability. In contrast, the derivative gain K_d enhances system damping, suppresses oscillations, and improves transient performance, although excessively large values may slow the system response and increase sensitivity to measurement noise.

The simulation results further indicate that no single PID parameter can simultaneously optimize all control performance indices. Instead, satisfactory control performance can only be achieved through the coordinated tuning of K_p , K_i , and K_d . The optimized PID parameters obtained through MATLAB/Simulink simulations enable the Furuta pendulum to reach equilibrium with a short settling time, low steady-state error, and reduced oscillation, demonstrating the effectiveness of parameter optimization in PID controller design.

A comparison with the LQR controller reveals that PID offers significant advantages in terms of implementation simplicity, ease of tuning, and practical applicability. Conversely, LQR provides smoother responses and superior stability by exploiting optimal state feedback, although it requires an accurate mathematical model and a more sophisticated design procedure. Consequently, for applications requiring rapid implementation or limited computational resources, the PID controller remains a practical and efficient solution.

The findings are consistent with previously published studies [3]–[6], which reported that increasing K_p improves response speed, K_i reduces steady-state error, and K_d enhances system stability. This agreement supports the validity of both the developed simulation model and the adopted research methodology.

Although promising results have been obtained, the present study is limited to numerical simulations in the MATLAB/Simulink environment. Future work will focus on experimental validation using a physical Furuta pendulum platform and the investigation of advanced control strategies, such as Adaptive PID, Fractional-Order PID, Fuzzy PID, and hybrid PID–LQR controllers, to further improve the performance of nonlinear control systems.

IV. Conclusions

This study developed a simulation model of the Furuta pendulum and systematically investigated the influence of the three PID controller parameters, namely K_p , K_i , and K_d , on the dynamic response of the system. MATLAB/Simulink simulations were conducted to clarify the role of each control component in terms of response speed, steady-state accuracy, and system stability.

The results indicate that the proportional gain K_p primarily determines the response speed but

may increase overshoot and oscillations when excessively large. The integral gain K_I effectively eliminates steady-state error but may prolong the settling process if improperly tuned. Meanwhile, the derivative gain K_d improves transient performance by suppressing oscillations, although excessive derivative action may reduce the response speed and increase sensitivity to measurement noise. The optimized PID parameters obtained through MATLAB/Simulink simulations enabled the Furuta pendulum to achieve a short settling time, low steady-state error, and reduced oscillation, demonstrating the effectiveness of parameter optimization in PID controller design.

A comparison with the LQR controller shows that the PID controller remains an attractive control strategy because of its simple structure, straightforward tuning procedure, and practical implementation, despite the superior dynamic performance generally achieved by LQR.

The main contribution of this work is the systematic and quantitative analysis of the individual effects of the PID parameters on the dynamic behavior of the Furuta pendulum, together with the identification of an effective parameter set for the investigated system. The findings provide useful guidance for the design and tuning of PID controllers in balancing systems and can be extended to applications such as self-balancing robots, stabilization devices, and precision control systems.

The present work is limited to numerical simulations. Future research will focus on experimental validation using a physical Furuta pendulum platform and on the development of advanced control strategies, including Adaptive PID, Fractional-Order PID, and hybrid PID–LQR controllers, to further enhance the control performance of nonlinear systems.

References

- [1] K. Stomach, “Standup and Stabilization of the Inverted Pendulum,” Univ. of California, 1999.
- [2] Caking, F. M. Bonsai, M. Tinker, “PID Control of Inverted Pendulum Using Adams and Matlab Co-Simulation,” Association for Computing Machinery, New York, USA, 2016.
- [3] Kaman Peres, PID Control of the Inverted Pendulum, *COMSCI Conf. Publication*, 2023
- [4] Bowen CSU, “A Comparative Study of PID and LQR Control Strategies Applied to Inverted Pendulum Systems,” Master of Engineering Thesis, University of Guelph, Canada, 2019.
- [5] R. Erotica, M. Duarte-Mermaid, C. Jauregui, G. Lanfranc, “Inverted pendulum stabilization by means of fractional order PID controllers,” *IEEE Conf. Publication*, 2017.
- [6] D. Chirme-Sisa, L. W. U. Mega, J. C. Herrera-Levant, R. J. Coquina-Castillo, “Modeling, Simulation, Design and Comparative Analysis of the PID and LQR Controllers for an Inverted Pendulum,” *IEEE Conf. Publication, IEEE Explore*, 2024.